Collaborations and Projects

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National collaborations

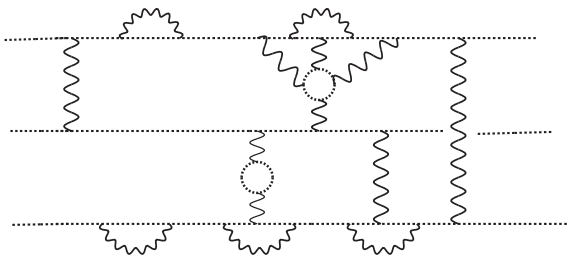
- San Luis Potosi: Victor M. Banda Guzmán (faculty).
- Queretaro: Adolfo Huet (faculty), Omar Fabian Izquierdo Perez (Lic. student of Adolfo).
- UNAM MC: Cesar Moctezuma Mata Zamora (postdoc with Manfred).
- Rochester, US: Misha Arturo Lopez-Lopez (postdoc with Antonino Di Piazza).
- Puebla: Jonathan Reyes Perez (hopefully postdoc with us after February 2027).
- My own students: Carlos J. Servin Tomas (PhD), Cristofer Nava Jacuinde (Lic.), Alondra Sanchez (Lic.).

QED perturbation theory in the worldline formalism

How to tame the proliferation of Feynman diagrams at high loops?

In QED: the worldline formalism can be used to combine all Feynman diagrams that differ only by the position of photon legs on scalar lines and loops

Feynman 1948, 1950, 1951: Representation of the perturbative QED S-matrix in terms of particle path integrals interconnected by photons in all possible ways:



Equivalent to Feynman diagrams, but a priori does not require fixing an ordering for the photon legs along a line or loop

Master formula for the N-photon amplitudes in scalar QED

Polyakov 1987 (in "Gauge Fields and Strings"), Bern and Kosower 1991, Strassler 1992:

$$\begin{aligned}\Gamma_{\rm scal}[\{k_i,\varepsilon_i\}] &= (-ie)^N \int_0^\infty \frac{dT}{T}(4\pi T)^{-\frac{D}{2}} e^{-m^2 T} \prod_{i=1}^N \int_0^T d\tau_i \\ &\quad \times \exp\Big\{ \sum_{i,j=1}^N \big[\frac{1}{2} G_{ij} k_i\cdot k_j + i\dot{G}_{ij} k_i\cdot\varepsilon_j + \frac{1}{2}\ddot{G}_{ij}\varepsilon_i\cdot\varepsilon_j\big]\Big\}\,|_{\rm lin(\varepsilon_1,\ldots,\varepsilon_N)}\end{aligned}$$

$$\begin{aligned} G(\tau_1,\tau_2) &= |\tau_1-\tau_2| - \frac{(\tau_1-\tau_2)^2}{T} \\ \dot{G}(\tau_1,\tau_2) &= \mathrm{sgn}(\tau_1-\tau_2) - 2\frac{(\tau_1-\tau_2)}{T} \\ \ddot{G}(\tau_1,\tau_2) &= 2\delta(\tau_1-\tau_2) - \frac{2}{T}\end{aligned}$$

T is the proper-time of the scalar particle in the loop, τ_i parametrizes the position of photon i along the loop.

Master formula for the N-photon amplitudes in spinor QED

A similar master formula can be written for the spinor loop using the worldline superfield

$$X^\mu \equiv x^\mu + \sqrt{2}\,\theta\psi^\mu$$

Polyakov 1987, Strassler 1992:

$$\begin{aligned}\Gamma_{\rm spin}[k_1,\varepsilon_1;\ldots;k_N,\varepsilon_N] \quad &= \quad -2(-ie)^N \int_0^\infty \frac{dT}{T}(4\pi T)^{-\frac{D}{2}} e^{-m^2 T} \\ &\times \prod_{i=1}^N \int_0^T d\tau_i \int d\theta_i \exp\left\{ \sum_{i,j=1}^N \left[\frac{1}{2}\hat{G}_{ij} k_i\cdot k_j + iD_i \hat{G}_{ij}\varepsilon_i \cdot k_j + \frac{1}{2} D_i D_j \hat{G}_{ij}\varepsilon_i\cdot\varepsilon_j \right] \right\} |_{\varepsilon_1\ldots\varepsilon_N}\end{aligned}$$

where the θ_i are Grassmann variables and

$$D_i = \frac{\partial}{\partial\theta_i} - \theta_i \frac{\partial}{\partial\tau_i}, \quad \hat{G}_{12} = G_{12} + \theta_1\theta_2 G_{F12}$$

$$G_F(\tau_1,\tau_2) \quad = \quad \mathrm{sgn}(\tau_1-\tau_2)$$

Advantages of the worldline representation in QED

1. It avoids the break-up of scalar/fermion lines or loops into individual propagators (important for external-field problems, where individual propagators are already complicated).
2. It yields the complete N-photon amplitudes, with no need to sum over photon permutations (not very relevant at the one-loop level, but becomes interesting at the multi-loop level).

Worldline representation of dressed electron propagator

Feynman 1951; Fradkin 1966; Fradkin and Gitman 1991; M. Reuter, M.G. Schmidt and C.S. 1996,
N. Ahmadiniaz, V.M. Banda Guzmán, F. Bastianelli, O. Corradini, J.P. Edwards and C. S. 2020, 2022:
Second-order representation of the x-space Dirac propagator $S^{xx'}[A]$ in a Maxwell background:

$$
\begin{aligned}
S^{xx'}[A] &= \left[m + i\not{D}'\right] K^{xx'}[A] \\
K^{xx'}[A] &= \left\langle x' \middle| \left[m^2 - D_\mu D^\mu + \frac{i}{2}\, e\gamma^\mu \gamma^\nu F_{\mu\nu}\right]^{-1} \middle| x \right\rangle \\
&= \int_0^\infty dT\ \mathrm{e}^{-m^2 T}\, \mathrm{e}^{-\frac{1}{4}\frac{(x-x')^2}{T}} \int_{q(0)=0}^{q(T)=0} Dq\ \mathrm{e}^{-\int_0^T d\tau \left(\frac{1}{4}\dot{q}^2 + ie\,\dot{q}\cdot A + ie\frac{x'-x}{T}\cdot A\right)} \\
&\quad \times 2^{-\frac{D}{2}}\, \mathrm{symb}^{-1} \int_{\psi(0)+\psi(T)=0} D\psi\ \mathrm{e}^{-\int_0^T d\tau \left[\frac{1}{2}\psi_\mu \dot{\psi}^\mu - ieF_{\mu\nu}(\psi+\eta)^\mu(\psi+\eta)^\nu\right]}
\end{aligned}
$$

Here η^μ is an external Grassmann Lorentz vector, and the "symbol map" *symb* converts products of ηs into fully antisymmetrised products of Dirac matrices:

$$
\mathrm{symb}\left(\gamma^{\alpha_1\alpha_2\cdots\alpha_n}\right) \equiv (-i\sqrt{2})^n \eta^{\alpha_1}\eta^{\alpha_2}\ldots\eta^{\alpha_n}
$$

where $\gamma^{\alpha\beta\cdots\rho}$ denotes the totally antisymmetrised product of gamma matrices:

$$
\gamma^{\alpha_1\alpha_2\cdots\alpha_n} \equiv \frac{1}{n!} \sum_{\pi\in S_n} \mathrm{sign}(\pi)\gamma^{\alpha_{\pi(1)}}\gamma^{\alpha_{\pi(2)}}\ldots\gamma^{\alpha_{\pi(n)}}
$$

NO LONG DIRAC PRODUCTS OR TRACES!

Ongoing projects

1. Nonlinear Compton (double Compton, triple Compton....) (Carlos, Cristofer).
2. Four-photon amplitudes, on-shell and off-shell (Victor, Cesar, Misha....)
3. Two-loop Scalar QED vacuum polarization (Victor...).
4. Numerical worldline Monte Carlo approach to one-loop N-point amplitudes (Adolfo, Omar).
5. Circular integration (Cesar,.....)
6. One-loop photon-graviton amplitudes, combining worldline and the perturbiner formalism.

Projects in planning

1. Nonlinear Compton in external fields (constant, plane wave).
2. Multiloop $g-2$.
3. Two-loop effective actions in the Standard Model (e.g. with W loop) (Jonathan....).
4. Three-loop Euler-Heisenberg Lagrangian.