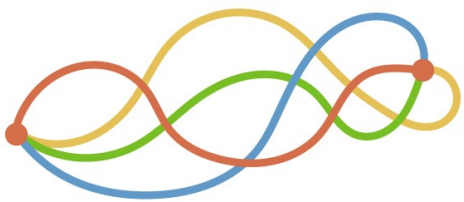
Scattering Amplitudes within the worldline formalism



Victor M. Banda

Universidad Politécnica de San Luis Potosí

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Academic and Research Journey

Background

PhD at IF, San Luis Potosí. Advisor: Mariana Kirchbach

Postdoc at IFM, Morelia with Christian Schubert

Part-time professor at San Luis Potosí

Research interest

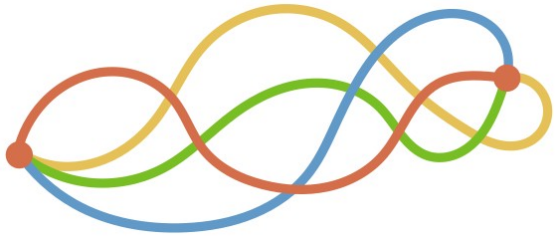
Development of analytical techniques for calculating scattering amplitudes using the worldline formalism

The worldline formalism is an alternative approach to quantum field theory based on first-quantized relativistic path integrals.

Example

Propagator of a relativistic scalar particle in a electromagnetic field A^μ

in Euclidean space-time



$$D^{xx'} = \int_0^\infty dT\, \mathrm{e}^{-m^2 T} \int_{x(0)=x}^{x(T)=x'} \mathcal{D}x\, \mathrm{e}^{-S}$$

$$S = \int_0^T d\tau \left[\frac{1}{4}\dot{x}^2(\tau) + ie\dot{x}(\tau)\cdot A(x)\right]$$

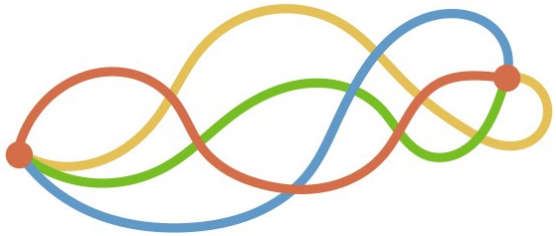
$\int \mathcal{D}x\, \mathrm{e}^{-S}$

Add contributions from every possible worldline connecting x with x'

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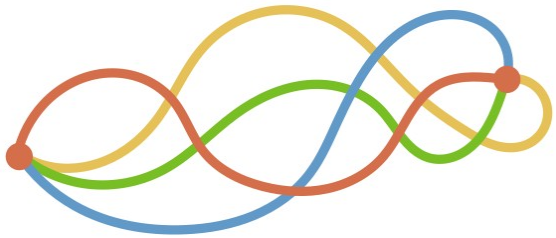
$\int_0^\infty dT$

Sum over all possible values of the particle's proper time.

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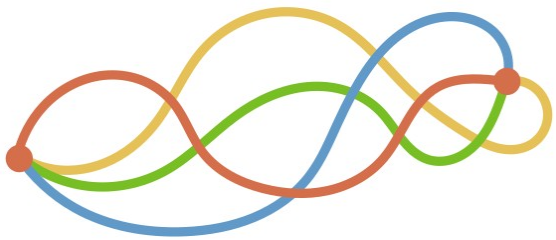
$\int_0^\infty dT\, \mathrm{e}^{-m^2 T}$

The exponential factor suppresses long worldlines.

Large mass m makes long worldlines unlikely.

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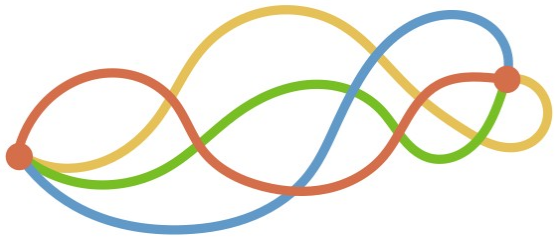
$\frac{1}{4}\dot{x}^2(\tau)$

The path integral favors smoother trajectories; extremely jagged paths are exponentially suppressed.

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in Euclidean space-time



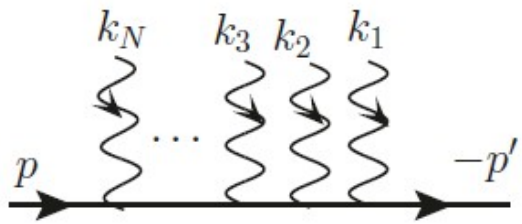
$$D^{xx'} = \int_0^\infty dT\, \mathrm{e}^{-m^2 T} \int_{x(0)=x}^{x(T)=x'} \mathcal{D}x\, \mathrm{e}^{-S}$$

$$S = \int_0^T d\tau \left[\frac{1}{4}\dot{x}^2(\tau) + ie\dot{x}(\tau)\cdot A(x)\right]$$

$ie\int_0^T d\tau\, \dot{x}\cdot A = ie\int_{\text{path}} A_\mu dx^\mu$

Every path accumulates an electromagnetic phase while moving through the gauge field.

During my postdoc we developed this formalism for the electron propagator dressed by an arbitrary number of photons

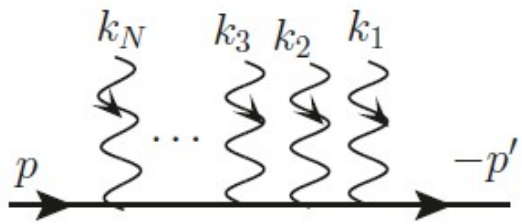


Off-shell. JHEP 08 (2020) 08, 049

On-shell. JHEP 01 (2022) 050

Work in progress: With an external electromagnetic background

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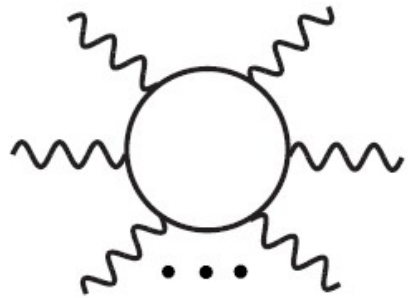
On-shell result

$$M^{pp'}_{Nss'} = \frac{(-ie)^N}{2m}\overline{u}_{s'}(p')K^{pp'}_N u_s(p)$$

$$K_N = A_N + B_{N\alpha\beta}\sigma^{\alpha\beta} - iC_N\gamma^5$$

Our current research also focuses on one-loop and multi-loop scattering amplitudes ...

N-photon one-loop amplitudes



Example for scalar QED

$$\Gamma_{\text{scal}}(k_1,\varepsilon_1;\ldots;k_N,\varepsilon_N) = (-ie)^N (2\pi)^D \delta^D\left(\sum k_i\right) \int_0^\infty \frac{dT}{T} (4\pi T)^{-\frac{D}{2}} e^{-m^2 T}$$

$$\times \prod_{i=1}^N \int_0^T d\tau_i \exp\left\{ \sum_{i,j=1}^N \left[\frac{1}{2} G_{ij} k_i \cdot k_j - i\dot{G}_{ij} \varepsilon_i \cdot k_j + \frac{1}{2} \ddot{G}_{ij} \varepsilon_i \cdot \varepsilon_j \right] \right\} \Big|_{\varepsilon_1 \varepsilon_2 \ldots \varepsilon_N} .$$

$$G_{ij} = |\tau_i - \tau_j| - \frac{(\tau_i - \tau_j)^2}{T}$$

We can construct multi-loop amplitudes

by sewing together external photon legs

From the one-loop four-photon amplitude and sewing two external photon legs we can obtain a worldline expression for the two-loop vacuum polarization

$$\Gamma^{(2)}_{\text{scal}}(\varepsilon_1,k_1;\varepsilon_2,k_2) = \left(-\frac{1}{2}\right)(-ie)^4 \int_0^\infty \frac{dT}{T} e^{-m^2T} \int_{x(0)=x(T)} Dx\, e^{-\int_0^T d\tau \frac{\dot{x}}{4}}$$
$$\times \int_0^\infty d\bar{T}(4\pi\bar{T})^{-\frac{D}{2}} \int_0^T d\tau_a d\tau_b d\tau_1 d\tau_2\, \varepsilon\cdot\dot{x}_1 \varepsilon\cdot\dot{x}_2\, e^{ik_1x_1+ik_2x_2}$$
$$\times\, \dot{x}_a\cdot\dot{x}_b\, e^{-\frac{(x_b-x_a)^2}{4\bar{T}}}\,.$$

$x_i = x(\tau_i)$

We can construct multi-loop amplitudes

by sewing together external photon legs

$$\Gamma^{(2)}_{\text{scal}}(\varepsilon,k_1;\varepsilon,k_2) = (2\pi)^D \delta(k_1+k_2)(\epsilon_1\cdot\epsilon_2 k^2 - \epsilon_1\cdot k_1 \epsilon_2\cdot k_2)\Pi^{(2)}_{\text{scal}}(k^2)$$

$$\begin{aligned}\Pi^{(2)}_{\text{scal}}(k^2) &= -\frac{e^4}{2(4\pi)^D}\int_0^\infty \frac{dT}{T^{1+\frac{D}{2}}}\,\mathsf{e}^{-m^2T}\int_0^\infty d\bar{T}\int_0^T d\tau_a\int_0^T d\tau_b\,\gamma^{\frac{D}{2}} \\ &\quad\times\int_0^T d\tau_1\int_0^T d\tau_2\; I\;\mathsf{e}^{-k^2\Delta}\end{aligned}$$

$$\begin{aligned} C &\equiv G_{1a} - G_{1b} - G_{2a} + G_{2b} \\ \Delta &= G_{12} - \frac{\gamma_{ab}C^2}{4}\end{aligned}$$

$$I = D\left(-\ddot{G}_{ab} + \frac{\gamma_{ab}}{2}\dot{G}^2_{ab}\right)\partial_1\Delta\partial_2\Delta - \frac{4}{\gamma^2_{ab}C^2}\partial_a\Delta\partial_b\Delta\partial_1\partial_2\Delta + \partial_2\left[\frac{4}{\gamma^2_{ab}C^2}\partial_a\Delta\partial_b\Delta\right]\partial_1\Delta$$

The challenge:

Develop mathematical methods to evaluate worldline master integrals

Some techniques developed so far

- Integration by parts (IBP) identities for worldline Green's functions.
- Identities relating bosonic and fermionic Green's functions and their derivatives.
- Evaluation of integrals with polynomial integrands involving bosonic and fermionic Green's functions and their derivatives.
- Discontinuity methods for one-loop on-shell worldline master formulas.

Ref.

PoS **LL2026**, 088 (2026), arXiv:2607.13321.

JHEP **07**, 159 (2025).

Phys. Rept. **355**, 73-234 (2001).

We still need more mathematical tools ...